5

POWER LAW ANALYSES

Power Law Analyses in General

Wind speed data are available in most cases for only one anemometer height — generally 10 meters above ground level. Since wind speed normally increases with height above the ground, and since most wind energy generation systems have hub heights of more than 10 meters, some means of estimating the wind speed at heights other than the measurement height is needed.

The two most common methods of estimating wind speed at hub height from measurements at a reference level are (1) to assume a logarithmic wind profile, and (2) to assume a power law. The empirically derived power law is used more often, although the logarithmic law is theoretically more sound since it is based on the principles of fluid mechanics.

To derive an expression for the logarithmic wind profile, Businger et al. (1971) and others have assumed that the variation of wind speed with height can be treated as a function of stability and surface roughness only. Drawbacks to the use of the logarithmic wind profile are the difficulties in measuring the friction velocity and the stability parameter.

Most investigators, therefore, have used a power law wind profile instead of the logarithmic profile. The power law is expressed as:

$$U_2 = U_1 \left(\frac{Z_2}{Z_1}\right)^{\alpha}$$

where:

U_2 = wind speed at height Z_2;
U_1 = wind speed at height Z_1;
Z_2 = computed height;
Z_1 = reference height;
α = power law exponent.

The power law exponent (α) is a function of surface roughness and stability. Empirical studies have found that an α value of 0.14 (1/7) best fits most sites. This method, therefore, is often referred to as the "one-seventh" power law.

Power Law Analyses of Montana Wind Data

GeoResearch, Inc., has performed power law analyses of wind data available from sites where monitoring was conducted at several heights. The purpose of these analyses was to compare application of the one-seventh law to use of the calculated power law exponent.

One year of data from the Livingston Candidate Wind Turbine site and 18 months of data from the Montana Power Company Salem site were analyzed. In addition, previously analyzed data from the Old West Regional Commission's monitoring site at Glasgow Air Force Base were examined and compared to the Livingston and Salem data.

The power law analyses involved calculating the power law exponent for each hour by means of the following equation:

$$\alpha = \frac{\log\left(\frac{U_{up}}{U_{low}}\right)}{\log\left(\frac{Z_{up}}{Z_{low}}\right)}$$

where:

α = power law exponent;
log = base 10 logarithm;
U = wind speed;
Z = anemometer height.

"Up" and "low" refer to the upper and lower anemometer heights, respectively.

The power law exponents were calculated for two different air layers: lowest to middle, and lowest to highest anemometer heights. Once all exponents were calculated, the monthly and yearly arithmetic mean and standard deviation of all

values for each hour were computed as follows:

$$\bar{\alpha} = \frac{1}{n} \sum_{i=1}^{n} \alpha_i$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} \alpha_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} \alpha_i\right)^2}{n-1}}$$

where:

$\bar{\alpha}$ = mean of α values;
σ = standard deviation;
n = number of α values.

Results of the analyses are listed in Tables V-1 through V-4. As a consequence of the great variation in wind speed over any given period, the standard deviations are quite large. Though not included in these tables, additional calculations showed that the average α values were highest during the nighttime hours and lowest during the afternoon for all months. This relationship was most pronounced during the summer.

An examination of the tables indicates that, at the Salem site, average monthly α values for the 10- to 30-meter anemometer heights ranged from 0.08 in December to 0.16 in July and November. The annual average value was 0.14. For the 10 - to 100 - meter heights, average monthly values of α ranged from 0.09 in December to 0.19 in November, with an annual average of 0.14.

At the Livingston site, average monthly α values for the 9.1 - to 30.0 - meter heights ranged from 0.07 in May to 0.39 in February. The annual average α value was 0.13. For the 9.1- to 45.7-meter heights, monthly average α values varied from 0.12 in December to 0.35 in February, with an annual average of 0.18.

Data from a wind monitoring site at Glasgow Air Force Base were previously analyzed to determine variation of wind speed with height by Pasquill stability category and by wind direction. Results of these analyses indicate average α values of about 0.10 for neutral and unstable conditions,

Table V-1
Power Law Exponents (Monthly Average)
Montana Power Company Salem Site
5/1/80 - 10/31/81

Month	Alpha 10.0-30.0m	Sigma 10.0-30.0m	Alpha 10.0-100.0m	Sigma 10.0-100.0m
January	0.12	0.22	0.18	0.21
February	0.14	0.15	0.16	0.14
March	0.12	0.24	0.13	0.19
April	0.14	0.15	0.14	0.16
May	0.13	0.19	0.12	0.16
June	0.15	0.17	0.16	0.14
July	0.16	0.33	0.15	0.21
August	0.13	0.19	0.12	0.17
September	0.13	0.19	0.13	0.18
October	0.15	0.24	0.17	0.19
November	0.16	0.17	0.19	0.16
December	0.08	0.25	0.09	0.17
Year	0.14	0.22	0.14	0.18

and from about 0.23 to 0.40 during stable conditions. Thus, applying the one-seventh law under unstable and neutral conditions would result in an overestimation of wind speeds aloft. Under stable conditions, however, the one-seventh law would underestimate actual wind speeds aloft. In general, stable and neutral conditions primarily occur during the period shortly before sunset until shortly after sunrise. Unstable conditions are more likely during the summer than in the winter.

The effect of using actual monthly power law exponents instead of the one-seventh exponent is

Table V-2
Power Law Exponents (Annual Average)
Montana Power Company Salem Site
5/1/80 - 10/31/81

Hour of Day	Alpha 10-30m	Sigma 10-30m	Alpha 10-100m	Sigma 10-100m
1	0.17	0.30	0.18	0.23
2	0.17	0.32	0.18	0.22
3	0.17	0.31	0.18	0.23
4	0.17	0.27	0.18	0.23
5	0.17	0.23	0.19	0.19
6	0.16	0.21	0.17	0.20
7	0.17	0.27	0.18	0.20
8	0.16	0.22	0.20	0.19
9	0.13	0.23	0.17	0.18
10	0.11	0.21	0.14	0.15
11	0.10	0.16	0.10	0.15
12	0.08	0.16	0.09	0.12
13	0.06	0.16	0.07	0.11
14	0.07	0.14	0.08	0.10
15	0.08	0.11	0.08	0.09
16	0.09	0.11	0.08	0.07
17	0.10	0.12	0.09	0.10
18	0.11	0.13	0.10	0.13
19	0.12	0.14	0.11	0.14
20	0.15	0.17	0.14	0.15
21	0.17	0.19	0.17	0.16
22	0.19	0.23	0.19	0.20
23	0.17	0.26	0.18	0.19
24	0.19	0.26	0.19	0.21
All Hours	0.14	0.22	0.14	0.18

shown for the Salem Site and the Livingston Candidate Wind Turbine Site in Tables V-5 through V-8. In these tables, the monthly averages of the measured wind speed readings at each anemometer height are given, together with the wind speed at that level computed from the actual average wind speed measured at the lowest height. Two power law exponents were used: the actual average power law exponent calculated from the data, and the one-seventh power law.

Use of the actual power law exponent and the one-seventh power law produced about the same

Table V-3
Power Law Exponents (Monthly Average)
Livingston Candidate Wind Turbine Site
9/1/80 - 6/30/82

Month	Alpha 9.1-30.0m	Sigma 9.1-30.0m	Alpha 9.1-45.7m	Sigma 9.1-45.7m
January	0.12	0.10	0.13	0.14
February	0.39	0.59	0.35	0.45
March	0.15	0.22	0.14	0.12
April	0.11	0.09	0.15	0.12
May	0.07	0.11	0.19	0.12
June	0.13	0.10	0.18	0.12
July	0.12	0.11	0.20	0.14
August	0.13	0.13	0.23	0.16
September	0.11	0.11	0.17	0.15
October	0.12	0.09	0.15	0.15
November	0.10	0.11	0.13	0.12
December	0.10	0.06	0.12	0.08
Year	0.13	0.21	0.18	0.19

level of accuracy. (See Tables V-9 and V-10.) The major exception was in February; however, the collected data at the lower height could be inaccurate. The one-seventh power law tended to be somewhat more accurate over longer than shorter periods.

Table V-4
Power Law Exponents (Annual Average)
Livingston Candidate Wind Turbine Site
9/1/80 - 6/30/82

Hour of Day	Alpha 9.1-30.0m	Sigma 9.1-30.0m	Alpha 9.1-45.7m	Sigma 9.1-45.7m
1	0.16	0.20	0.20	0.19
2	0.15	0.21	0.20	0.19
3	0.15	0.20	0.19	0.19
4	0.15	0.21	0.19	0.20
5	0.14	0.22	0.18	0.21
6	0.15	0.21	0.19	0.21
7	0.14	0.22	0.18	0.21
8	0.15	0.21	0.18	0.19
9	0.12	0.21	0.17	0.19
10	0.12	0.25	0.15	0.17
11	0.09	0.16	0.14	0.14
12	0.11	0.25	0.15	0.20
13	0.11	0.23	0.15	0.19
14	0.10	0.23	0.14	0.18
15	0.11	0.20	0.14	0.16
16	0.11	0.22	0.15	0.18
17	0.12	0.19	0.15	0.16
18	0.12	0.17	0.16	0.15
19	0.13	0.15	0.17	0.14
20	0.15	0.18	0.19	0.16
21	0.16	0.20	0.20	0.18
22	0.15	0.21	0.20	0.19
23	0.17	0.22	0.21	0.20
24	0.17	0.22	0.21	0.20
All Hours	0.13	0.21	0.18	0.19

Table V-5

Actual vs Calculated Wind Speeds

Montana Power Company Salem Site - 30 Meters

(miles per hour)

Month	Actual	Actual Exponent	1/7 Power Law
January	9.4	9.1	9.4
February	16.1	16.1	16.1
March	9.6	9.8	10.1
April	15.0	15.2	15.2
May	10.2	10.0	10.2
June	12.0	11.9	11.8
July	10.7	10.7	10.5
August	9.1	9.0	9.1
September	11.4	11.3	11.5
October	12.9	12.9	12.8
November	16.2	16.2	15.9
December	16.3	15.6	16.7
Year	11.9	11.9	11.9

Table V-6

Actual vs Calculated Wind Speed

Montana Power Company Salem Site - 100 Meters

(miles per hour)

Month	Actual	Actual Exponent	1/7 Power Law
January	12.7	12.1	11.1
February	20.3	19.9	19.2
March	12.1	11.6	11.9
April	18.1	17.9	18.1
May	11.8	11.5	12.1
June	14.6	14.6	14.0
July	12.7	12.7	12.5
August	10.7	10.3	10.8
September	13.8	13.2	13.6
October	16.4	16.1	15.1
November	20.8	21.1	18.9
December	17.5	17.6	19.9
Year	14.5	14.1	14.2

Table V-7
Actual vs Calculated Wind Speeds
Livingston Candidate Wind Turbine Site - 30 Meters
(miles per hour)

Month	Actual	Actual Exponent	1/7 Power Law
January	20.0	20.4	21.0
February	25.4	33.0	24.5
March	19.2	15.1	14.9
April	16.7	16.9	17.5
May	12.1	12.0	13.0
June	13.6	13.8	14.0
July	13.8	12.7	14.1
August	11.1	11.1	11.3
September	13.0	12.9	13.4
October	17.3	17.4	17.9
November	22.4	21.7	22.9
December	26.2	26.3	27.6
Year	18.1	18.2	18.5

Table V-8
Actual vs Calculated Wind Speeds
Livingston Candidate Wind Turbine Site - 45.7 Meters
(miles per hour)

Month	Actual	Actual Exponent	1/7 Power Law
January	22.7	21.8	22.3
February	26.5	36.4	26.1
March	20.5	15.8	15.9
April	18.2	18.9	18.6
May	14.4	14.9	13.9
June	15.3	15.8	14.9
July	15.7	16.4	15.0
August	13.2	13.8	12.0
September	14.7	14.9	14.2
October	19.0	19.2	19.0
November	22.8	23.8	24.3
December	27.4	28.3	29.3
Year	19.5	20.9	19.6

Table V-9
Accuracy of Power Law Exponents
Montana Power Company Salem Site
(Percentage Difference From Actual Speed)

Month	30 Meters		100 Meters	
	Actual Exponent	1/7 Power Law	Actual Exponent	1/7 Power Law
January	-3	—	-5	-13
February	—	—	-2	-5
March	2	5	-4	-2
April	1	1	-1	—
May	-2	—	-3	3
June	-1	-2	—	-4
July	—	-2	—	-2
August	-1	—	-4	1
September	-1	1	-4	-1
October	—	-1	-2	-8
November	—	-2	1	-9
December	-4	2	1	14
Year	—	—	-3	-2

Table V-10
Accuracy of Power Law Exponents
Livingston Candidate Wind Turbine Site
(Percentage Difference From Actual Speed)

Month	30 Meters		45.7 Meters	
	Actual Exponent	1/7 Power Law	Actual Exponent	1/7 Power Law
January	2	5	-4	-2
February	30	-4	37	-2
March	-21	-22	-23	-22
April	1	5	4	2
May	-1	7	3	-3
June	1	3	3	-3
July	-1	2	4	-4
August	—	2	5	-9
September	-1	3	1	-3
October	1	3	1	—
November	-3	2	4	7
December	—	5	3	7
Year	1	2	7	1